

# Analytical model of the pressure variation in the gerotor pump chambers

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## Abstract

In this paper the operating characteristics of the gerotor pump with fixed gear axis are described. Gerotors are used as motors, compressors and expanders, as well as in the new ecological and economical star rotor engine. The subject of the investigation is the internal combustion engines lubricating pump and the possibilities for the new construction solutions with better characteristic with the aim of increasing the pump energy efficiency. The basic objective of this investigation is the definition of the relation between the single geometrical parameters and the values of the pressure variation in the pump chambers as the consequence of the change of the chamber's current volume. On the basis of the geometrical-kinematical model of the pump gear profile, the determination formula of the chamber's current volume and the corresponding pressure variation are defined in this paper. The derived analytical formulae and graphical interpretation of the obtained results which are given in the paper open the possibility for the analysis of the influence of the gearing geometrical parameters on the variation of the instantaneous pressure. The obtained results can be used for the calculation of the precise values of the forces and torques affecting to the gear pair of gerotor pump, as well as its volumetric losses. The objective of the solution choice is obtaining the best design of the gerotor pump with minimum energy losses.

**Key words:** gerotor, trochoidal gearing, instantaneous volume, pressure variation

## 1. Introduction

The basic demands which are made on the pumps of the hydraulic systems are to ensure the necessary fluid flow and pressure with durability at the pump's minimum total weight and volume. In the construction process of the pumps it is necessary that the influence of the numerous different

pump parameters to its output characteristics must be considered. Therefore, the base of the investigation in this paper would be to identify the influence of the change in the geometrical parameters' profile of the gerotor pumps' working elements to their operating characteristics. Due to the significant advantages of the trochoidal gearing, the characteristics of trochoid and their conjugate envelopes have been investigated by a number of scientists.

Ansdale and Lockley [1] derived closed-form equations of the geometry for two types of conjugate envelope. Those authors demonstrated the value of the existing closed-form equations in the design of a Wankel rotary engine. Colbourne [2] defined eight types of conjugate envelope for each trochoid where the number of envelope lobes was either bigger or smaller by one than the number of trochoid lobes. The same author [3] described a method for calculating the tooth contact stresses in internal gear pumps, and it has been shown that a considerable reduction can be achieved in the maximum contact stress by altering the proportions commonly used in existing pumps. Robinson and Lyon [4] were able to modify the equations by introducing a constant which accounts for the space that is required in the sealing design. Maiti and Sinha [5] developed a kinematic analysis that has been carried out to investigate the pattern of rolling and sliding in the load transmitting contact regions. The authors presented a generalized method developed to find out and analyze the flow rate, ripple, and speed variation in different kinematic models [6]. Maiti [7] presents the theoretical guidelines for selecting the inlet-outlet port sizes, their position and sequences of the flow distributor valves used in epitrochoid generated rotary piston machine type of hydrostatic units, which have been established in the presented analysis. Beard et al. [8] derived relationships which show the influence of the trochoid ratio, the pin size ratio, and the radius of the generating pin on the curvature

of the epitrochoidal gerotor. Shung and Pennock [9] present unified and compact equations describing the geometric properties of the different types of trochoid and conjugate envelope. They present a simplified analytical model of a trochoidal-type machine in which friction and deformation at the contact points are neglected [10]. Mimmi and Pennacchi, [11] with a general method were showing the analytical condition for avoiding undercutting by using the concept of the limit curve. Mancò et al. [12] present a general procedure for the computerized design of gerotor lubricating pumps for internal combustion engines. Vecchiato et al. [13] have developed the geometry of rotor conjugated profiles applying the theory of envelopes to a family of parametric curves and analysis of profile meshing. They discussed the determination of singularities and computerized design of pumps with rotor profiles free of singularities. Paffoni [14] used a vector analysis and derived equations for defining the precise geometry of a gear pump using non-conventional profile. From this analysis, speed, normal force and pressure is deduced in analytical closed form. Kim et al. [15] define the geometry of the rotors starting from the design parameters and showed the process of choosing the solution which is subject to some limitations in order to limit the pressure angle between the rotors. Those authors consider the design optimization.

The previously described investigations start from the hypothesis that the pressure in all chambers of one zone is the same, in other words, the influence of the chamber volume variation on the pressure variation in the gerotor pump chamber is not considered. For this reason, the former theoretical analysis in the area of the trochoidal gearing taken into consideration, the basic aim of this paper would be to develop a mathematical model of the operating characteristics of the pump with internal trochoidal gearing and simulation of the current pressure variation in the pump chambers. For the calculation of the current area of the pump chamber section, the method described in reference [5] is applied and for the calculation of the pressure variation in the pump chambers the modification of method described in reference [10], [16] is used.

## 2. Geometrical and kinematical model

Gearing of the trochoidal pump's gear pair where the external gear has one tooth more than the internal gear is considered in this paper. The profile of the internal gear is described with equidistance of epitrochoide and the profile of the external gear is described by the circular arc with radius  $r_c$ . Meshing of all the teeth is carried out simultaneously in the trochoidal gear pairs with theoretical profiles. For this reason, it is necessary to derive general equations of the profiles' points' coordinates, which are applicable to all teeth. To derive the coordinate equations in any one of the contact points, it was necessary to carry out the generalization of the geometrical relations between the angles of the trochoidal gear pair elements rotation. The model of kinematic pair with stationary axis of the gears is accepted, with the driving shaft fixed to the internal gear [17].

The basic geometrical and kinematic relations for generating the equidistance of the epitrochoide and its conjugate envelope used for defining gearing profile of the examined gerotor pump are shown in Figure 1.

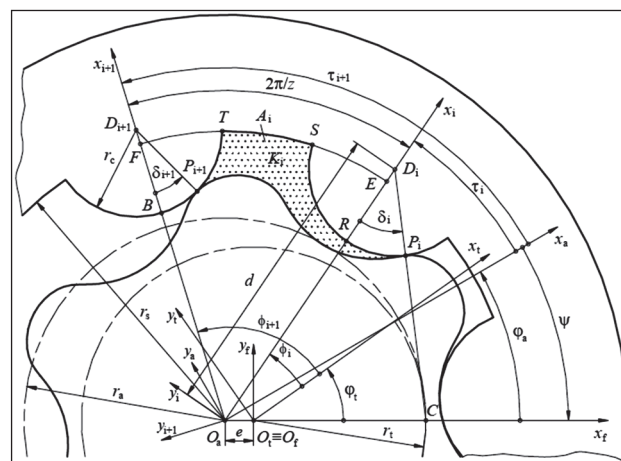


Figure 1. Schematic presentation of the gerotor pump gear pair with basic geometrical dimensions

Different coordinate systems have been introduced to derive the equations for the profile points, and these systems are described in detail in reference [18]. During the relative motion of the kinematic circles with radius  $r_a$   $r_i$  while the point  $D_i$  is generating an epitrochoid, the point  $P_i$  is generating the equidistance. The angle indicated  $\phi_i$  is the rotation angle of the trochoid coordinate system and

$\delta_i$  is the leaning angle. The number of the external gear teeth is indicated with  $z$  and it corresponds to the number of the pump chambers. Position vector of the contact point  $P_i$  in the coordinate system of trochoid can be defined in matrix form as:

$$\vec{r}_{P_i}^{(t)} = \begin{bmatrix} e[\cos z\varphi_i + \lambda z \cos \varphi_i - c \cos(\varphi_i + \delta_i)] \\ e[\sin z\varphi_i + \lambda z \sin \varphi_i - c \sin(\varphi_i + \delta_i)] \\ 1 \end{bmatrix} \dots (1)$$

In equation (1)  $\lambda$  is the trochoid coefficient,  $\lambda=d/ez$ , where  $c$  is the equidistant coefficient,  $c=r/e$ .

On the basis of the geometrical relations shown in the Figure 1, it is possible to determine the angle  $\delta_i$  as

$$\delta_i = \arctan \frac{\sin(z-1)\varphi_i}{\lambda + \cos(z-1)\varphi_i} \dots\dots\dots (2)$$

If the angle  $\psi$  formed by the axis  $x_a$  and  $x_f$  is taken as referent rotating angle, it is necessary to express the angle  $\phi_i$  in the function of the angle  $\psi$

$$\varphi_i = \tau_i + \frac{\psi}{z-1}, \dots\dots\dots (3)$$

where  $\tau_i$  is the angle between the axe  $x_a$  and the axe  $x_i$

$$\tau_i = \frac{\pi(2i-1)}{z} \dots\dots\dots (4)$$

Analogous to that,

$$\tau_i = \frac{\pi(2i-1)}{z} \dots\dots\dots (5)$$

and

$$\varphi_{i+1} = \frac{\pi(2i+1)}{z} + \frac{\psi}{z-1} \dots\dots\dots (6)$$

After the definition of the gerotor pump gearing geometry and the forming of the basic kinematical model, it is possible to determine the operating characteristics of the pump.

### 3. Analysis of the pressure variation in the pump chamber

Conventional calculation methods of the pump load start from the hypothesis that the pressure in all the chambers of the same zone (inlet and delivery) is constant. In this case, the force of fluid pressure, which separates the inlet zone from the delivery zone, is a continual force that can be presented as an equivalent concentrated pressure force [19]. During the working process of the pump in every working chamber, due to fluid flow, the pressure is changing. It means that for the modeling of pump gear loading, the pressure in each of the chambers must be known at any time. In addition, as the consequence of the presence of production tolerance, specific geometry and working conditions of the pump, the fluid leaking appears with the direct influence to the volume losses. The fluid leaking out through the gap between the teeth profiles is the consequence of the pressure difference between the two neighboring chambers. For precise calculation of pressure increase or decrease between the two neighboring working pump chambers, pressure variations in the pump chambers must be determined. At the meshing simulation, numbering of the contact points is necessary, as well as the numbering of all the teeth, and it must be established which teeth are in meshing. During that, the teeth of the external gear are indicated with  $i = 1, 2, \dots, z$ , while internal gear teeth are signed as  $j = 1, 2, \dots, z-1$ . The pump chambers are indicated with  $K_i$  and teeth contact points with  $P_i$  (Figure 2).

When the pumps with the stationary shaft axis are considered, the fluid distribution is done by sickle form holes in the housing, as shown in the Figure 2, which means that the open space area for the fluid flow is changeable. With such constructions, the open space area for the fluid flow is the same as the current chamber area. Starting from the given hypothesis and the energy preservation law Mancò S. at al. [12, 16] have defined the formula for the calculation of the pressure variation during the fluid flow in the chamber  $K_i$  in the following form:

$$\Delta p_i = \frac{p_f b^2}{A_i^2} \left[ \frac{dA_i}{dt} \right]^2, \dots\dots\dots (7)$$

where the  $\rho_f$  is fluid density,  $b$  is gear thickness,  $A_i$  is the current chamber area and  $dA/dt$  is area variation of chamber  $K_i$ .

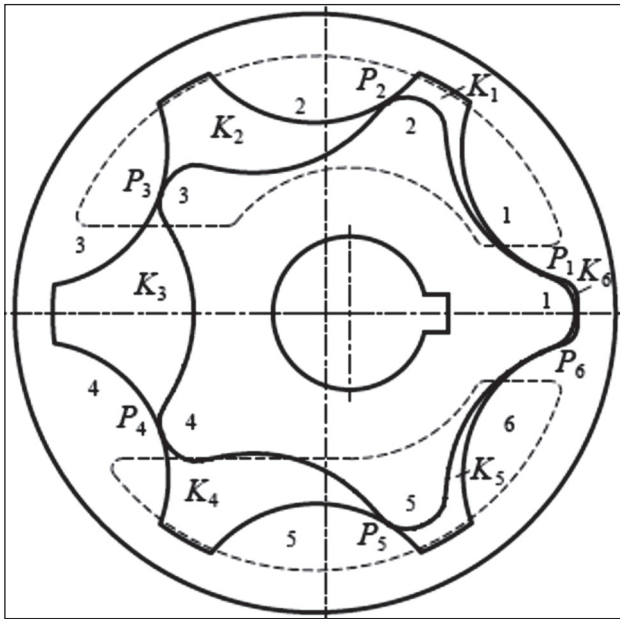


Figure 2. Schematic presentation of the gerotor pump with sickle distribution holes

The pressure change in the inlet chamber can be derived accordingly, the following formula:

$$p_i = p_u - \Delta p_i \quad p_i = p_u - \Delta p_i, \dots \dots \dots (8)$$

and for the outlet chamber

$$p_i = p_p + \Delta p_i, \dots \dots \dots (9)$$

During the inlet phase, the chamber volume increases and the pressure decreases, while, in the delivery phase, the chamber volume decreases and the pressure increases.

#### 4. Analysis of the chamber volume variation of the gerotor pump

The volume variation of the chamber during the working process of the gerotor pump will be considered. With the gerotor pumps, due to specific geometry of the gearing profile, continual contact of all the teeth is realized, which enables the necessary separation of the zones of the high and low pressure in the working area of the pump. The working chambers are the space between the profiles of the external and internal gear, and during working, the teeth have the role of the pushing elements (pistons), while the chambers correspond to cylinders. The chambers' volume is periodically increased and decreased, and they are in turn related to the inlet and outlet lead. To calculate the instantaneous area variation of the pump working chamber, the method presented in the reference [5] can be used. In the Figure 1 the geometrical relations to the determination of the area  $A_i(\psi)$  for the kinematic model of a pump with the fixed axis are given. The requested area  $A_i$  can be calculated according to the following equation:

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$$A_i = S_a - S_t + S_1 - S_2, \dots \dots \dots (10)$$

where are:  $S_a$  is the segment of area limited with the envelope profile,  $S_t$  is the segment of area limited with the profile of trochoid,  $S_1$  is the area of triangle and it is equal to:

$$S_1 = \frac{1}{2} e^2 [\lambda z \sin(\tau_i - \psi) - c \sin(\tau_i - \psi + \delta_i)] \dots \dots \dots (11)$$

The area of triangle is nominated as  $S_2$  and it is equal to:

$$S_2 = \frac{1}{2} e^2 [\lambda z \sin(\tau_{i+1} - \psi) - c \sin(\tau_{i+1} - \psi + \delta_{i+1})] \dots \dots \dots (12)$$

The area  $S_a$  is considered as the sum of the geometrical elements area, calculated as follows

$$S_a = S_3 + S_4 - S_5 - S_6 + S_7 - 2S_8 + 2S_9 - 2S_{10}, \dots \dots \dots (13)$$

where  $S_3$  is the area of the circular section

$$S_3 = \frac{r_s^2 \pi}{z} \dots \dots \dots (14)$$

The area of triangle is nominated as  $S_4$  and it is calculated as

$$S_4 = \frac{1}{2} e^2 \lambda c \sin \delta_i \dots \dots \dots (15)$$

The area of triangle is nominated as  $S_5$  and it is equal to

$$S_5 = \frac{1}{2} e^2 \lambda c \sin \delta_{i+1} \dots \dots \dots (16)$$

The area of the circular section is nominated as  $S_6$  and is calculated as

$$S_6 = \frac{1}{2} e^2 c^2 \delta_i \dots\dots\dots (17)$$

The area of the circular section is marked as  $S_7$  and is equal to

$$S_7 = \frac{1}{2} e^2 c^2 \delta_{i+1} \dots\dots\dots (18)$$

$S_8$  is the area of the circular section, which is equal to the area of the circular section

$$S_8 = \frac{1}{2} e^2 c^2 \arccos \frac{c^2 + \lambda^2 z^2 - (r_s/e)^2}{2c\lambda z} \dots\dots\dots (19)$$

$S_9$  is the area of triangle, which is equal to the area of triangle

$$S_9 = \frac{1}{2} e^2 c \lambda z \left\{ 1 - \left[ \frac{c^2 + \lambda^2 z^2 - (r_s/e)^2}{2c\lambda z} \right]^2 \right\}^{\frac{1}{2}} \dots\dots\dots (20)$$

$S_{10}$  is the area of the circular section, which is equal to the area of the circular section

$$S_{10} = \frac{1}{2} r_s^2 \arccos \frac{(r_s/e)^2 + \lambda^2 z^2 - c^2}{2\lambda z (r_s/e)} \dots\dots\dots (21)$$

The area  $S_t$  is calculated as the area of the section which has been marked with the curve line defined in the parameter form as

$$S_t = \frac{1}{2} \int_{\varphi_i}^{\varphi_{i+1}} \left( x_t \frac{dy_t}{d\varphi} - y_t \frac{dx_t}{d\varphi} \right) d\varphi, \dots\dots\dots (22)$$

with the coordinates of the trochoidal profile points defined with the equations (1). Starting from the equations (1), by their differentiating and putting in order of the formula under the integral, the following can be derived:

$$S_t = \frac{1}{2} e^2 \int_{\varphi_i}^{\varphi_{i+1}} f(\varphi) d\varphi, \dots\dots\dots (23)$$

where is

$$f(\varphi) = z + \lambda^2 z^2 + c^2(1 + \delta') - c\lambda z(2 + \delta') \cos \delta + \lambda z(z + 1) \cos(z - 1)\varphi - c(z + 1 + \delta') \cos[(z - 1)\varphi - \delta] \dots\dots\dots (24)$$

By the integral, formula is obtained:

$$S_t = e^2 \pi \left( 1 + \lambda^2 z + \frac{c^2}{z} \right) + \frac{1}{2} e^2 \left[ c^2 \delta - \sin \delta + \frac{\lambda z(z + 1)}{(z - 1)} \sin(z - 1)\varphi \right] \Big|_{\varphi_i}^{\varphi_{i+1}} - \frac{1}{2} e^2 c \int_{\varphi_i}^{\varphi_{i+1}} f_1(\varphi) d\varphi \dots\dots\dots (25)$$

where is:

$$f_1(\varphi) = 2\lambda z \cos \delta + (z + 1 + \delta') \cos[(z - 1)\varphi - \delta] \dots\dots\dots (26)$$

Relation between the angles  $\phi_i$  and  $\psi$  has been established using the formula (3), and certain transformations have been made, formula for calculation the current area is obtained:

$$A_i = e^2 \left\langle \frac{\pi}{z} (s^2 - z - \lambda^2 z^2 - c^2) - c^2 \arccos \frac{c^2 + \lambda^2 z^2 - s^2}{2c\lambda z} + c\lambda z \left\{ 1 - \left[ \frac{c^2 + \lambda^2 z^2 - s^2}{2c\lambda z} \right]^2 \right\}^{\frac{1}{2}} \right\rangle - r_s^2 \arccos \frac{s^2 + \lambda^2 z^2 - c^2}{2\lambda z s} - \frac{\lambda z^2 e^2}{z - 1} \sin(z - 1)\varphi \Big|_{\varphi_i}^{\varphi_{i+1}} + c e^2 z \int_{\varphi_i}^{\varphi_{i+1}} \left[ 1 + \lambda^2 + 2\lambda \cos(z - 1)\varphi \right]^{\frac{1}{2}} d\varphi, \dots\dots\dots (27)$$

where is  $s = r_s/e$ .

When the differential is done and expressed in the function of the referent angle  $\psi$ , calculation of chamber' area variation is obtained:

$$\frac{dA_i}{dt} = \omega_1 e^2 z \left\{ 2\lambda \sin \frac{\pi}{z} \sin \left( \frac{2\pi i}{z} - \psi \right) - \frac{c}{z} \left[ 1 + \lambda^2 - 2\lambda \cos(\tau - \psi) \right]^{\frac{1}{2}} \right\} \frac{1}{r_i} \dots\dots\dots (28)$$

As well as that of the flow rate

$$Q = b \sum_{i=m}^n \frac{dA_i}{dt} \dots\dots\dots (29)$$

where  $m, n$  are indexes of the beginning and final chambers which can be found at the same time in the thrust phase. With the substitution of the equations (27) and (28) in (7) it is possible to calculate the pressure variation in the pump chambers.

**5. Simulation of the pressure variation in the pump chambers**

The developed mathematical model has been tested on the gear pairs of the pump models. One of previously defined demands is external loading, defined through the values of working pressure of 0.6 MPa (6 bar) and pump volume of  $14 \cdot 10^{-6} \text{ m}^3/\text{rev}$ . The geometrical characteristics which remain constant, yet obtained with the necessary overall dimensions of the gear pair are:  $b$  gear width,  $e$  eccentricity and the radius of the external gear's root circle expressed by  $r_s$ . The number of teeth of the outer gear  $z$  is constant. It is necessary to determine the values of coefficients  $\lambda$  and  $c$ , which define the optimal form of teeth profile. Two different gear sets are taken into consideration, commercial one and the other gear set has the profile form obtained on the basis of calculation given in the reference [19]. Geometrical parameters of the gear sets are:  $z=6, e=3.56 \text{ mm}, b=16.46 \text{ mm}, r_s=26.94 \text{ mm}$ . For gear set GP-375  $\lambda=1.375, c=2.75$  and for gear set GP-575  $\lambda=1.575, c=3.95$ . Other characteristics are:  $\Delta p=0.6 \text{ MPa}, \rho_f=900 \text{ kg/m}^3, n_1=1500 \text{ rpm}, \omega_1 = 2\pi n_1 = 50\pi \text{ s}^{-1}$ .

On the base of the previously conducted analysis, in the Figure 3, a diagram is shown of the instantaneous chamber area, and in the Figure 4, chamber area variation is presented. The reference angle is the rotation angle of the exterior gear,  $\phi_a$ . The figures clearly show that the considered

chamber  $K_i$  in position  $\phi_a=120^\circ$  has the greatest volume, the inlet phase is finished and is coming to the delivery phase. After the angle rotation of  $180^\circ$  in the chamber, the delivery phase is finished, the chamber has minimum volume and is coming to the inlet phase.

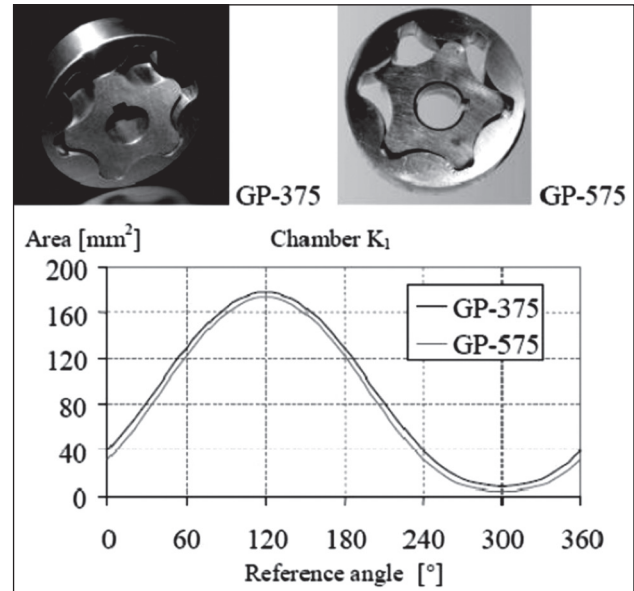


Figure 3. Diagram of the chamber area for both gear sets

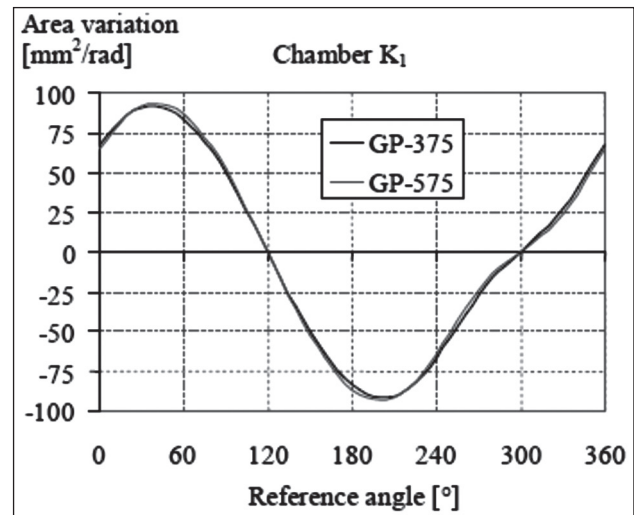


Figure 4. Diagram of the chamber area variations for both gear sets

In the Figure 5 the diagrams of flow rate of the gerotor pumps in relation of the rotation angle of the external gear are shown. On the basis of the geometrical interpretation of the pump flow rate with the same parameter  $r_s$ , it can be concluded that in the pumps with the same number of chambers, there are minor differences in the flow pulsation.

The Figure 6 shows the graphical interpretation of the pressure variation in the pump chambers in relation of the rotation angle, for the gear sets with the different coefficient  $\lambda$ . Through the mutual comparison of the diagrams a conclusion can be made that with the increase of the coefficient  $\lambda$  the value of the pressure change in the chambers is also rising. The volume degree of the pump performance is greater with the model that has a smaller value of the coefficient  $\lambda$ , smaller pressure changes in the pump chambers (Figure 6), and thus the smaller volume losses.

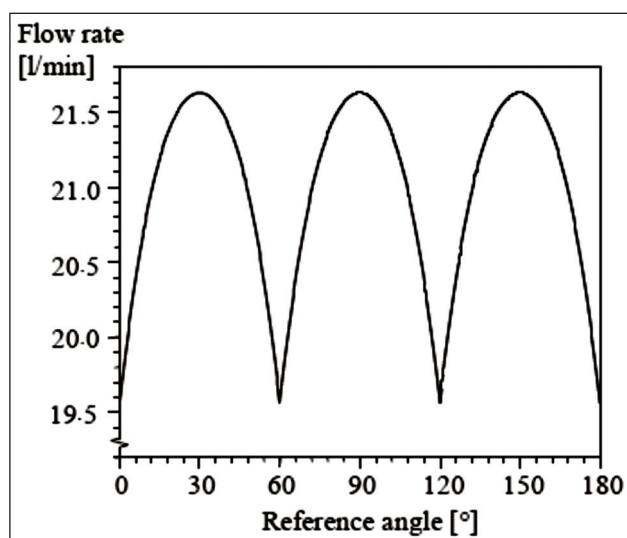


Figure 5. Diagram of the flow rate for gear pumps with the both gear sets

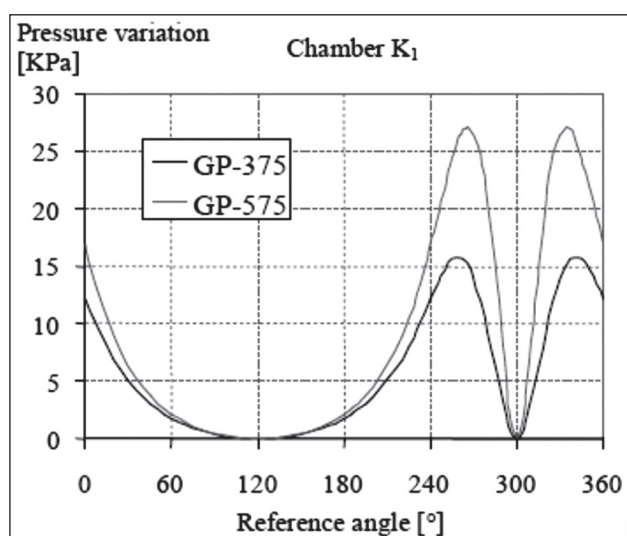
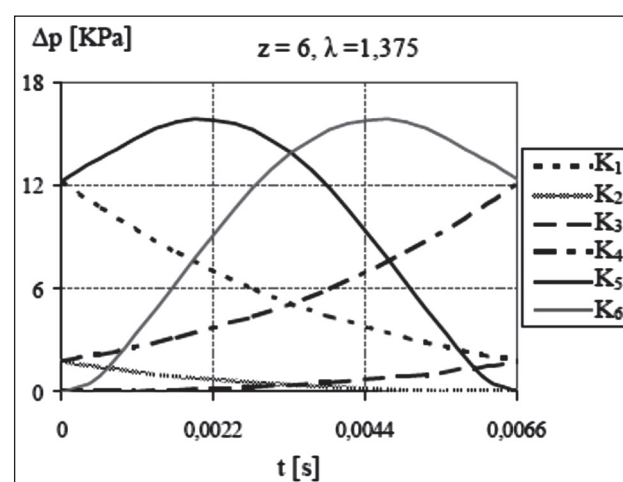


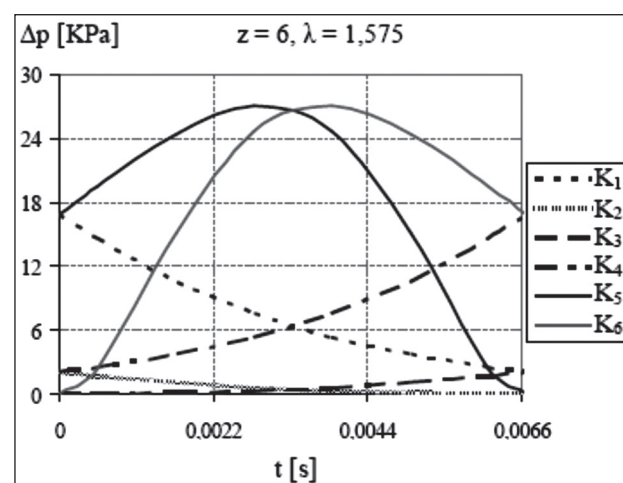
Figure 6. Diagram of the pressure variations in chamber  $K_1$  for both gear sets

With the aim of better consideration of the pressure variation in the pump chambers, as well as to

establish characteristic phases in the pump working cycles, in the Figure 7 the parallel diagrams for all the chambers and two different gear sets are given. In relation to the starting position, presented in the Figure 2, the chambers  $K_6$ ,  $K_1$  and  $K_2$  are in the inlet phase, and the chambers  $K_3$ ,  $K_4$  and  $K_5$  are in the delivery phase. On the basis of the given graphical interpretation, it can be concluded that the greatest difference in the pressures between the neighboring chambers is realized in the finish of the delivery phase and on the entrance into the delivery phase (between chambers  $K_5$  and  $K_6$ ), and thus the influence of the pressure change on the volume losses will be the greatest in these chambers.



a) gear set GP-375



b) gear set GP-575

Figure 7. Diagrams of the pressure variation in the pump chambers for both gear sets

The examinations of the two considered models of gear sets have been completed, with the simulation of the real operating conditions of the

pumps, and the results of the measuring are given in the Figure 8.

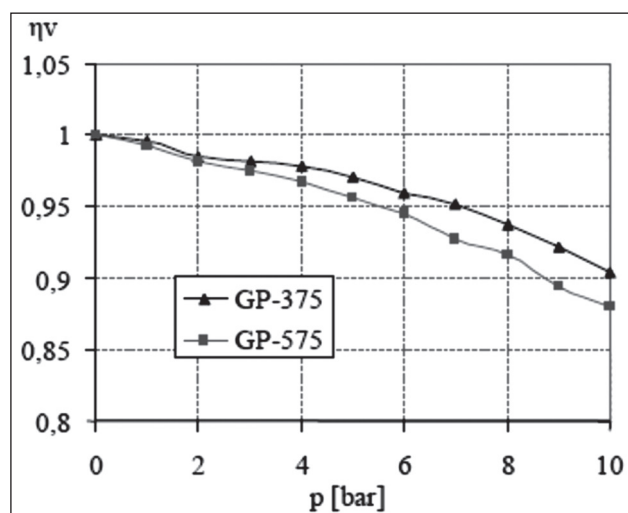


Figure 8. Diagram of volumetric efficiency

With the aim of illustrating the gear sets' geometry influence on the volume degree performance change, measuring of the pump with different teeth geometry gear sets built in has been carried out. From the energetic efficiency aspect, the adequate choice of a pump with a higher performance level is of a great importance in the industrial use.

## 6. Conclusion

The aim of this investigation was to define certain relations between the operating characteristics of the trochoidal gear pairs, through the theoretical consideration such as the flow rate, pressure variation in the pump chambers and the values of the considered geometrical parameters. It has been shown that, by the gerotor pump with the same number of chambers and the same radius of the root circle, the choice of the smaller values of the trochoid coefficient is changing the form of the gearing profile, but it does not significantly change the pump flow. It has also been shown that, with the new profile form, smaller pressure variation in the pump chambers is realized. The developed mathematical model and the obtained results can be of use to the constructors of the gerotor pump and motors for choosing the best constructive solutions that reaches higher coefficient of efficiency.

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